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THE USE OF A PARITY BIT IN A TELEMETRY SYSTEM

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GODDARD SPACE FLIGHT CENTER
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ABSTRACT

The use of parity in a pulsecode modulated (PCM) telemetry system is described. Parity performance curves are shown for several word lengths when the signal is in a noisy channel. Advantages of using parity over uncoded PCM words is shown to be 2.4 to 2.6 db. Other advantages are listed.

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ILLUSTRATIONS

THE USE OF PARITY BIT IN A TELEMETRY SYSTEM

INTRODUCTION

Parity has been used in some telemetry systems and has long been used in digital computers to detect errors in the transmission of binary data. Parity is the addition of an extra bit to each word to make the number of ones in a word either odd or even. In telemetry systems, an odd number is usually selected in order to provide more bit transitions and hence better synchronization. Although even parity has equal error-detection capabilities, odd parity is assumed throughout this document. When a word is sent, the parity bit ensures an odd number of ones in the word. Advantages of using parity are:

- It is the simplest error-detection code to implement.
- It may be checked at many points in the telemetry system to give a measure of total system performance up to that point.
- It results in a lower residual-word-error rate after the detected incorrect words are discarded.
- The residual words in error have at least two bits in error (all single bit errors are detected). Consequently, it is quite likely that the word in error will differ markedly from neighboring measurements and may be discarded by inspection.

ANALYSIS

In this study, the lack of a correct parity check means that the word is rejected. Rejected words are those having an odd number of bit errors (i.e., one error, three errors, five errors, etc.). Because single errors have the highest probability of occurrence, there is a sizeable detection of words having errors. Therefore, when a correct parity check occurs, the remaining words have a higher probability of being correct. Undetected word errors are those words which contain an even number of bit errors. These words have a much lower probability of occurrence at the signal-to-noise ratios (SNR) normally used in telemetry systems. SNR is defined as energy per bit per noise spectral density (ST/No) with white gaussian noise.

In order to provide a reference, the error rate for an uncoded word is calculated first. That is,

$$p_w = 1 - (1 - p_e)^N,$$

where

N = number of bits per word

p_e = probability of bit error in a binary symmetric channel.

For comparison of a telemetry system with or without parity, the word rate has been kept constant. Thus, when parity is used, there is less energy per bit because there is one more bit per word period. There is, therefore, a slightly higher bit-error rate. However, this loss is more than offset by the detection of word errors by the parity check. The method used is to calculate the probability of each odd number of bit errors per word by applying the expression for the binomial distribution. For a bit-error probability of p_e, the probability of having k errors in a word of n bits is

$$p_w(k, n) = \frac{n!}{k!\,(n-k)!}\, p_e^{\,k}\,(1 - p_e)^{n-k}$$

A bit-error probability (p_e) is selected from the standard bit-error rate curve (Figure 1). An SNR degradation of 10 log N-1/N (db) is used when selecting p_e for a system with parity. Then the above calculation is performed for all possible odd k's in the word. Addition of these numbers gives the discarded word ratio (DWR = p (1, n) + p (3, n) + . . .). Except for very low SNR it is only necessary to calculate the first two terms because the remaining terms are insignificant. At high SNR the first term dominates; at these SNR's parity can be considered to be a single bit-error encoding-detection scheme. The new word-error rate is calculated by subtracting the above sum from the total word errors received and correcting for the drop-in word quantity by dividing by 1-DWR. That is,

$$p_w \text{ (with parity)} = \frac{\text{TWR} - \text{DWR}}{1 - \text{DWR}}$$

where TWR = total word-error rate = p (1, n) + p (2, n) + p (3, n) + . . .

Figure 2 shows the calculated results for 5-bit data words (6 bits including parity). In this figure the performance of a parity coded word is compared to the uncoded word. In addition, the word acceptance/rejection characteristic is shown. For example, at SNR = 4 db, 90 percent of the words are accepted (10 percent are rejected for parity failure) and the word-error rate in the accepted words is 6.4×10^{-3} . This compares to a word-error rate of 60×10^{-3} with no parity error detection (all other things being equal). At high SNR's, parity does even better because fewer double errors will be made. Figures 2 through 7 show the calculated results for 5- through 10-bit data words.

In interpreting these graphs, one might be tempted to compare only the two error-rate curves at a constant word-error rate. The conclusions could then be reached that a savings in transmitter power (of the SNR difference between the coded and uncoded curves) could be achieved by using parity. This description is incomplete because the word rejection effect has not been included in the comparison. For example, in Figure 6, for a transmitter power reduction of 2.6 db at uncoded $p_w = 10^{-3}$, the number of rejected words increases from 0.22 percent to 4.6 percent. If the transmitted power were not reduced, the word rejection would remain 0.22 percent and the residual word-error rate in the accepted 99.78 percent would be $p_w = 2.8 \times 10^{-6}$. A change in SNR cannot be made without considering both p_w and word rejection.

The foregoing analysis treats the properties of pulsecode modulation (PCM) with parity in the presence of Gaussian white noise and random bit errors. A second class of errors are caused by radio frequency (RF) multipath fades, impulse interference, and tape recorder dropouts which may be classed as non-Gaussian and nonwhite. In general, these errors result in a burst of mistakes instead of a random assortment of single bit errors. The following paragraphs discuss parity as applied to tape recorders with their dropout errors and RF multipath fade errors. The conclusions may be easily extended to other types of errors.

Tape recorders form a necessary part of today's telemetry systems when data is to be stored for future processing. In general, a tape recorder adds very little noise to the signal being recorded (SNR greater than 30 db), but small imperfections in the tape may cause the loss of some number of consecutive bits. This loss is referred to as a dropout. Representative dropout statistics are not available, but it is reasonable to conclude that single bit errors will be more frequent than double bit errors and that triple bit errors will be more frequent than quadruple bit errors. Therefore, more words will be in error by an odd number of bits than by an even number of bits. Consequently, parity checks will detect at least one-half of the word errors caused by dropouts.

During a short RF multipath fade (less than a one-word period) much of the foregoing paragraph applies. However, during long fades, many consecutive words may be in error. Under these low-signal, high-noise conditions, parity will have a high chance of being wrong and up to 50 percent of the bad words will be detected. This represents a very high instantaneous word-error rate. Computer processing routines could easily recognize this fact and skip over or flag the entire portion of high instantaneous error rate. This technique would throw out some potentially good data, but the portion retained would be of higher quality.

IMPLEMENTATION

Figure 8 is a block diagram of a circuit which generates a parity check. Its timing and waveform are shown in Figure 9 for the case of a 5-bit data word. In addition to the normal encoder (running at the higher bit rate of n/n-1), there is a flip-flop and gating circuit. Data are shifted out of the data register and into a parity counter. The state of the counter during the last information bit in the word determines the parity bit state to be transmitted. The gating circuit first allows data bits to be passed to the transmitter. At the start of the last bit of the word, the gating circuit allows the parity bit to be placed in the serial bit stream, completing the word. The counter is reset and the gating circuitry shifts back to the next data word and the process repeats.

Power drain for the circuit shown in Figure 8 may be as little as 13 mw, a very small addition when compared to the power used in most PCM encoders. The few added components, being standard modules, will not significantly degrade reliability.

At the receiving end, the check for parity can be made either by hardware or in a computer. Because the hardware is simple, parity is usually checked in this way. Most PCM decoding equipment is now built with a parity check capability.

CONCLUSION

When it is desired to transmit measurements with higher confidence, and when slightly fewer measurements (because of discarded measurements) per time are permitted, much can be gained by the use of parity. In nonparity PCM systems, almost the same number of words <u>should</u> be thrown out but there is no way to tell the right words from the wrong ones. Usually, no new or additional equipment is needed at the receiving end of the telemetry link, and only a few components are required by the encoder. The use of parity is an inexpensive way of getting better telemetry performance.

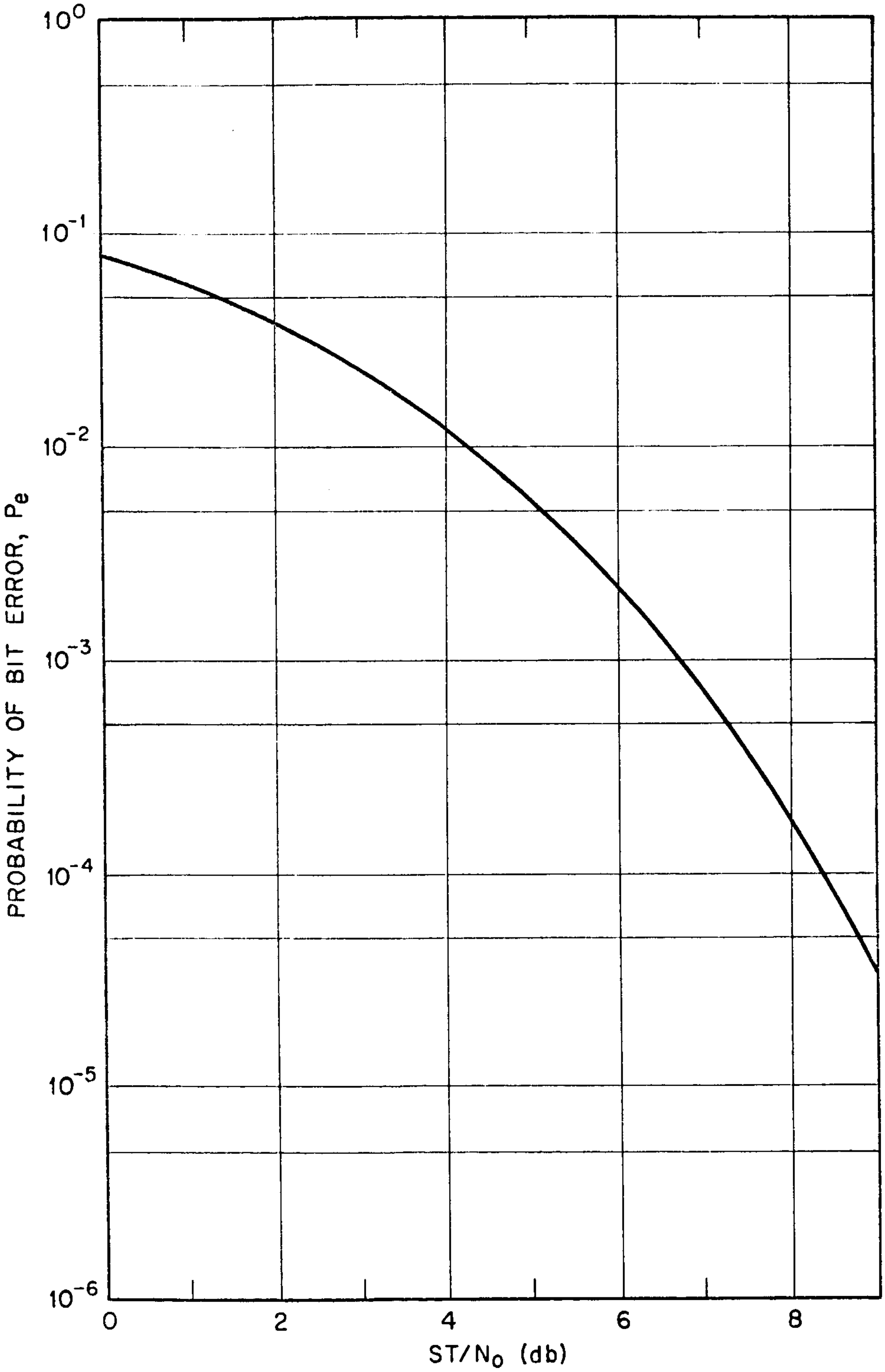

Figure 1. Probability of Bit Error vs SNR

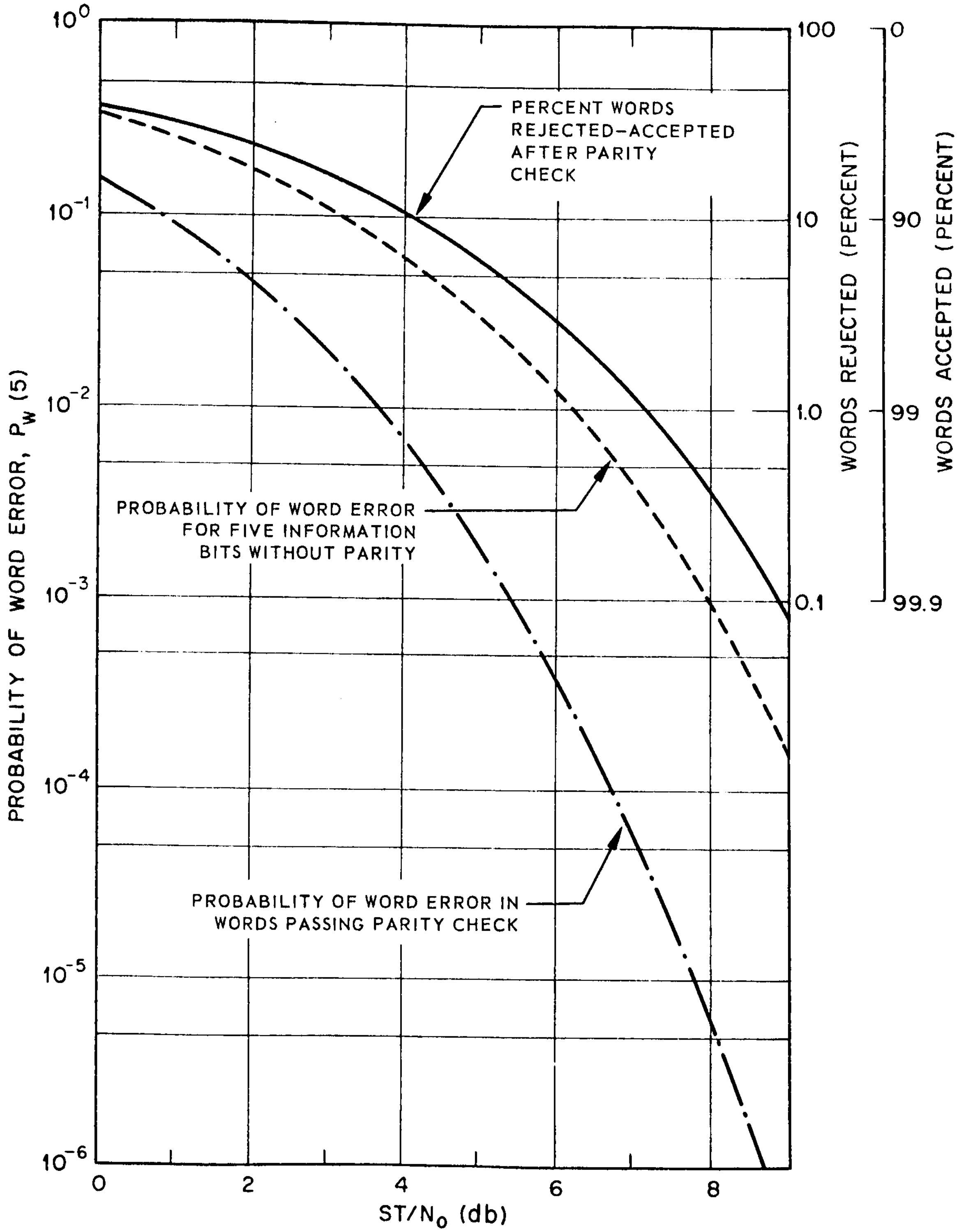

Figure 2. Parity Performance Curves for Five Bits Per Word

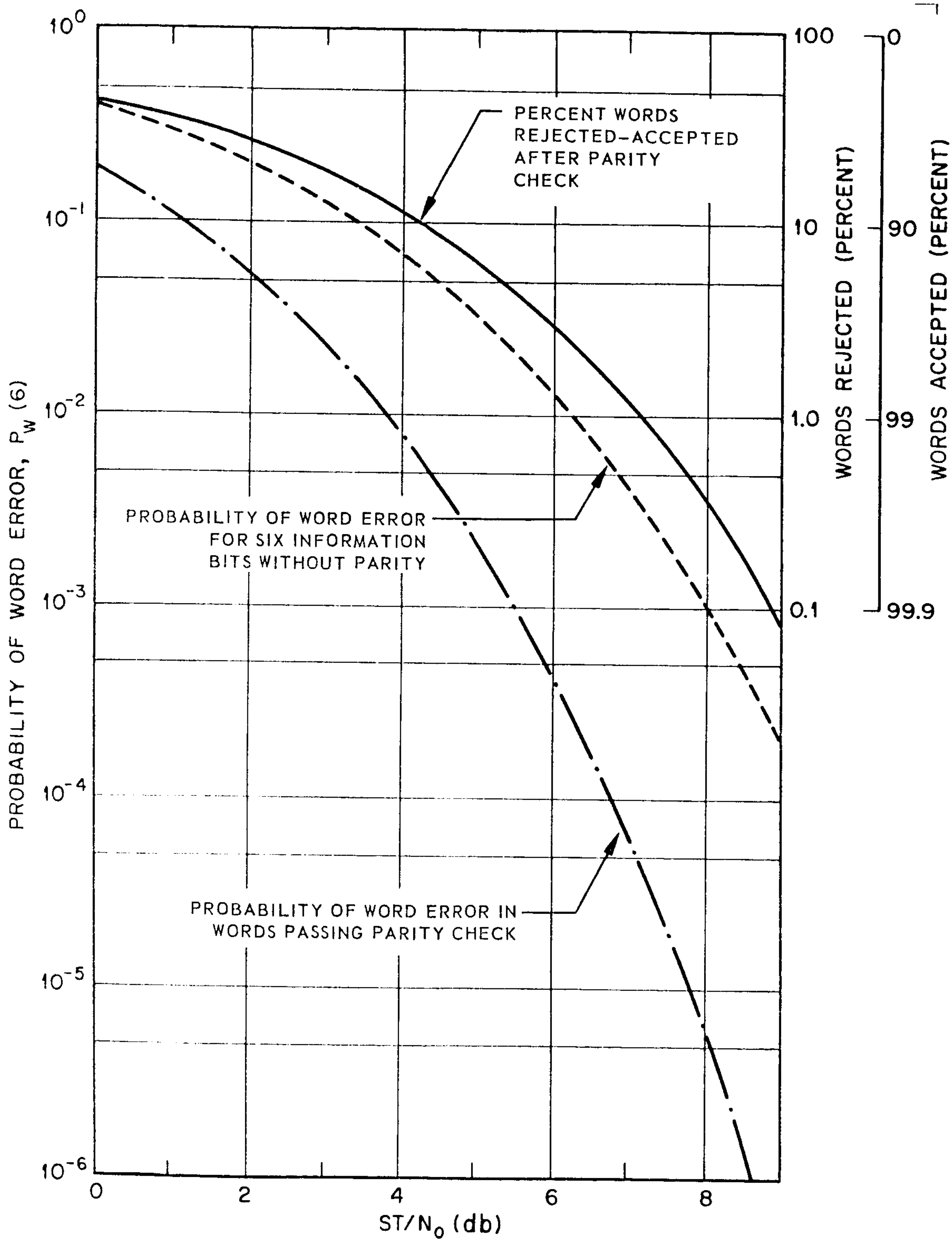

Figure 3. Parity Performance Curves for Six Bits Per Word

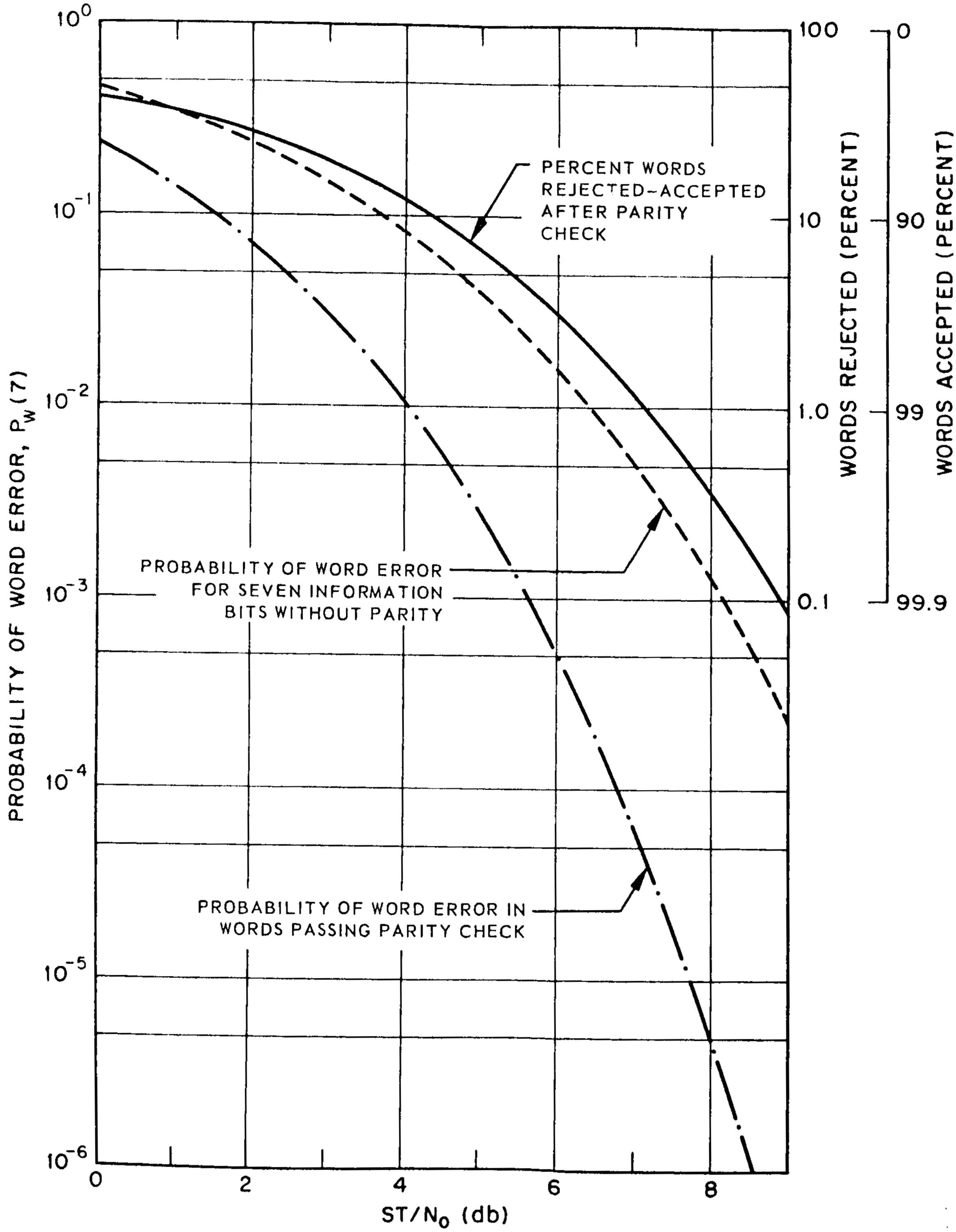

Figure 4. Parity Performance Curves for Seven Bits Per Word

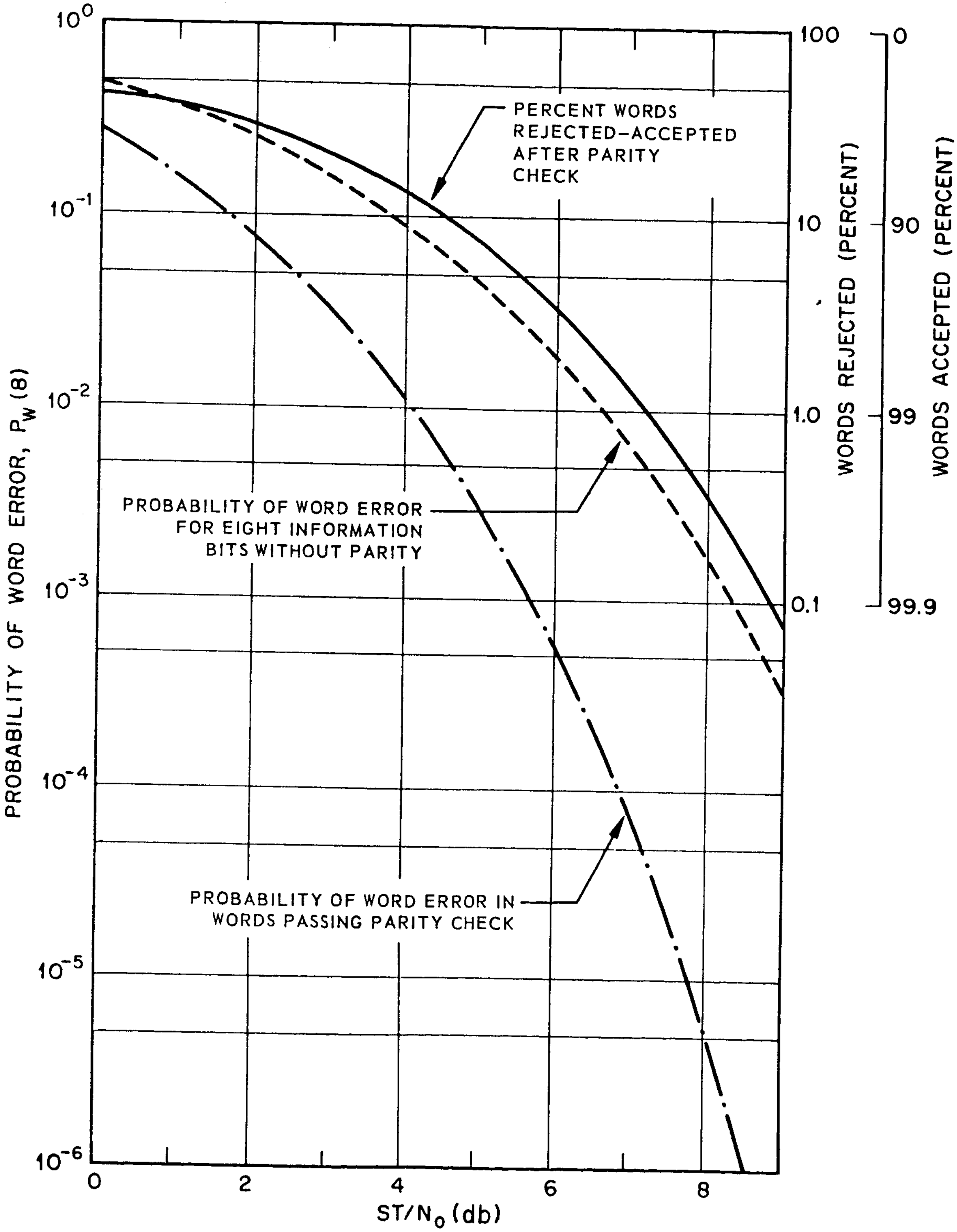

Figure 5. Parity Performance Curves for Eight Bits Per Word

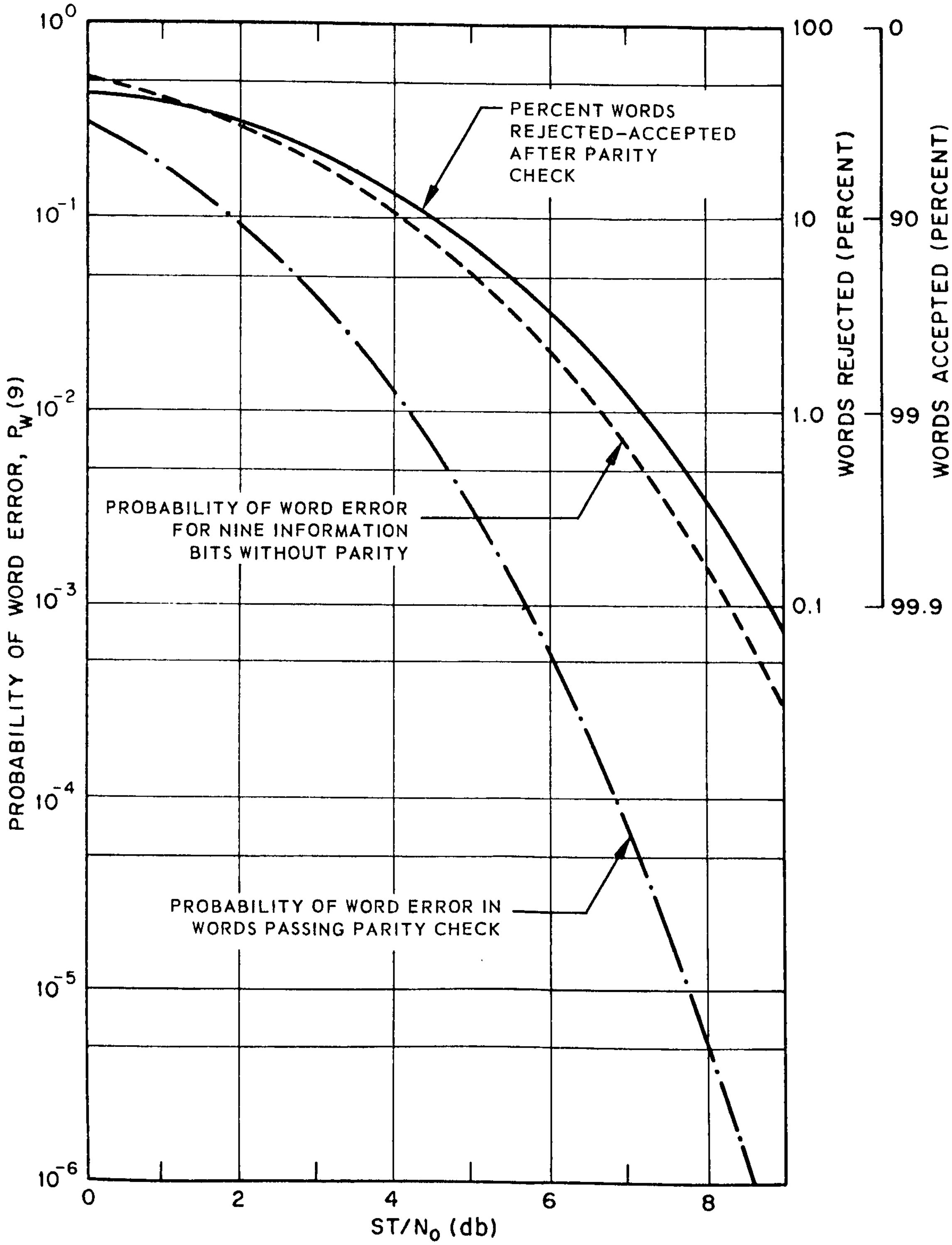

Figure 6. Parity Performance Curves for Nine Bits Per Word

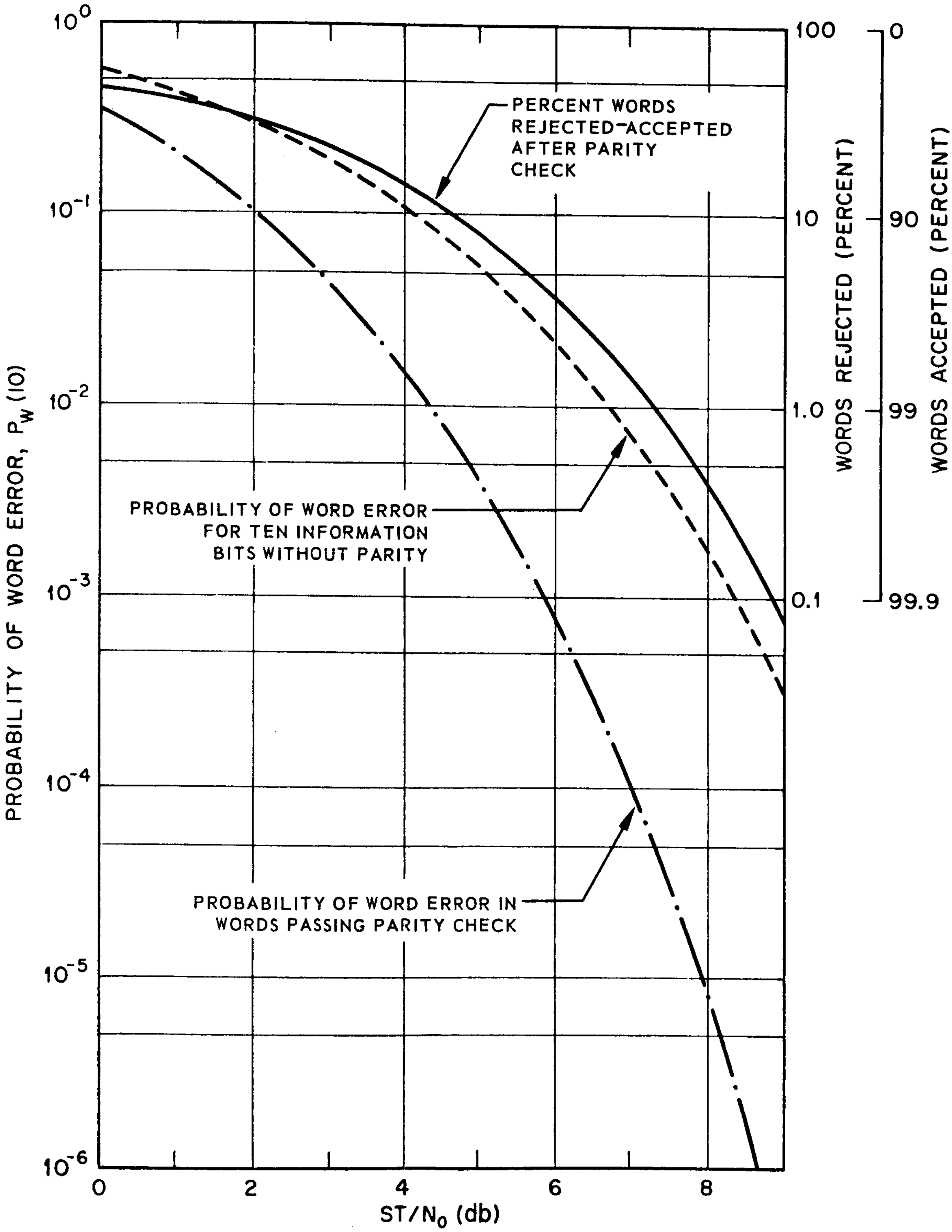

Figure 7. Parity Performance Curves for Ten Bits Per Word

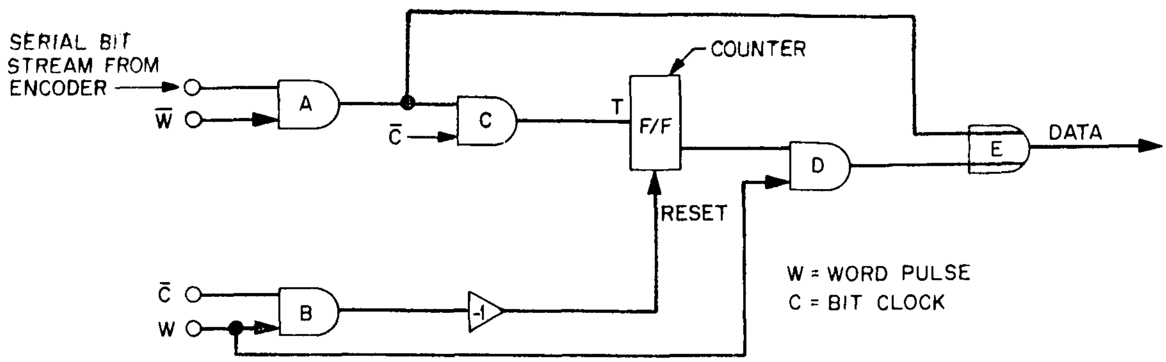

Figure 8. Parity Circuitry, Block Diagram

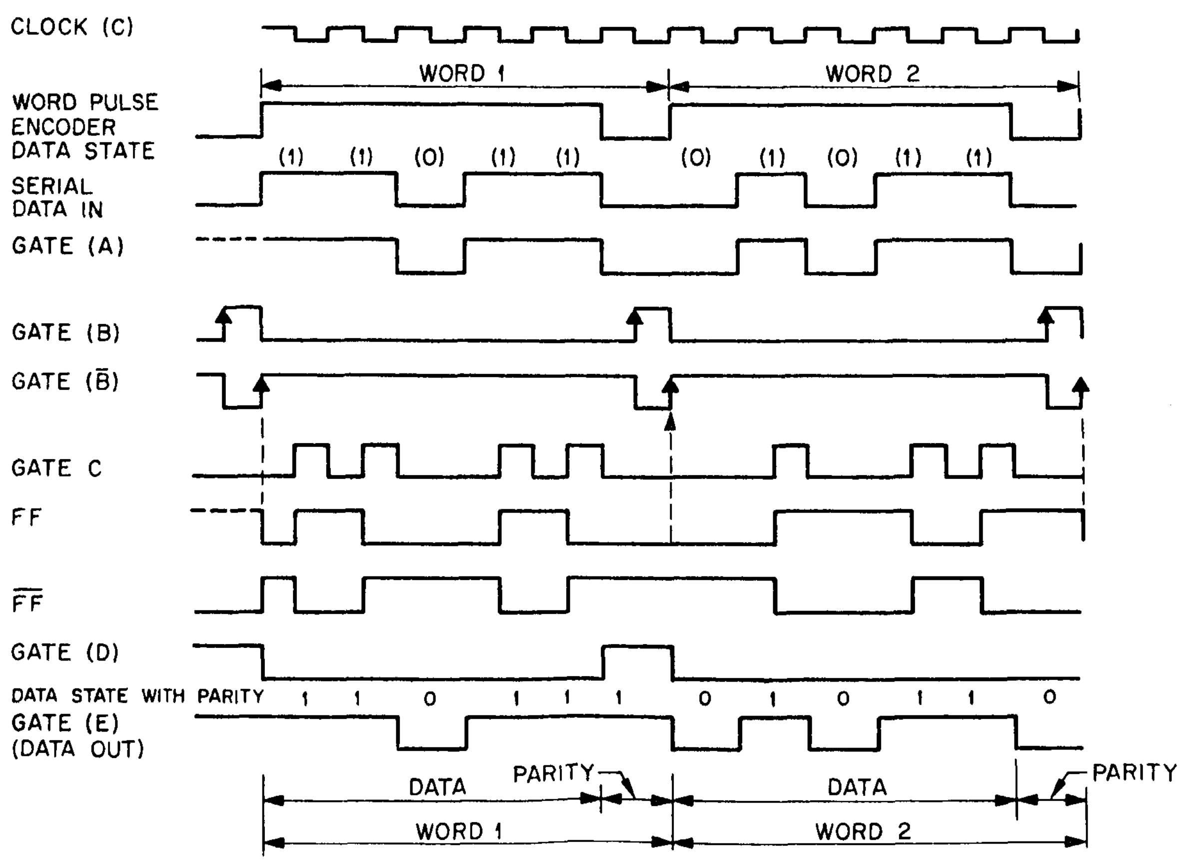

Figure 9. Parity Timing Diagram